6 Exponential and Logarithm Functions

In this chapter we are going to look at exponential and logarithm functions. Both of these functions are very important and need to be understood by anyone who is going on to later math courses. These functions also have applications in science, engineering, and business to name a few areas. In fact, these functions can show up in just about any field that uses even a small degree of mathematics.

Many students find both of these functions, especially logarithm functions, difficult to deal with. This is probably because they are so different from any of the other functions that they've looked at to this point and logarithms use a notation that will be new to almost everyone in an algebra class. However, you will find that once you get past the notation and start to understand some of their properties they really aren't too bad. So, we'll make sure to go over the notation and various properties of both exponential and logarithm functions in the hope that you will agree that once you understand those they aren't too bad.

In addition, we'll take a look at solving equations with exponentials and solving equations with logarithms. We'll also take a quick look a couple of applications involving exponentials and logarithms.

The following sections are the practice problems (without solutions) for this material.

If you are looking for the solutions to these problems you can go to the Practice Problems on the website and download a pdf with the solutions from there.

6.1 Exponential Functions

1. Given the function $f(x) = 4^x$ evaluate each of the following.

   (a) $f(-2)$ **(b)** $f\left(-\frac{1}{2}\right)$ **(c)** $f(0)$ **(d)** $f(1)$ **(e)** $f\left(\frac{3}{2}\right)$

2. Given the function $f(x) = \left(\frac{1}{5}\right)^x$ evaluate each of the following.

   (a) $f(-3)$ **(b)** $f(-1)$ **(c)** $f(0)$ **(d)** $f(2)$ **(e)** $f(3)$

3. Sketch each of the following.

   (a) $f(x) = 6^x$ **(b)** $g(x) = 6^x - 9$ **(c)** $g(x) = 6^{x+1}$

4. Sketch the graph of $f(x) = \mathbf{e}^{-x}$.

5. Sketch the graph of $f(x) = \mathbf{e}^{x-3} + 6$.

6.2 Logarithm Functions

For problems 1 - 3 write the expression in logarithmic form.

1. $7^5 = 16807$
2. $16^{\frac{3}{4}} = 8$
3. $\left(\frac{1}{3}\right)^{-2} = 9$

For problems 4 - 6 write the expression in exponential form.

4. $\log_2 32 = 5$
5. $\log_{\frac{1}{5}} \frac{1}{625} = 4$
6. $\log_9 \frac{1}{81} = -2$

For problems 7 - 12 determine the exact value of each of the following without using a calculator.

7. $\log_3 81$
8. $\log_5 125$
9. $\log_2 \frac{1}{8}$
10. $\log_{\frac{1}{4}} 16$
11. $\ln \mathbf{e}^4$
12. $\log \frac{1}{100}$

For problems 13 - 15 write each of the following in terms of simpler logarithms

13. $\log\left(3x^4y^{-7}\right)$
14. $\ln\left(x\sqrt{y^2+z^2}\right)$
15. $\log_4\left(\frac{x-4}{y^2\sqrt[5]{z}}\right)$

For problems 16 - 18 combine each of the following into a single logarithm with a coefficient of one.

16. $2\log_4 x + 5\log_4 y - \frac{1}{2}\log_4 z$
17. $3\ln(t+5) - 4\ln(t) - 2\ln(s-1)$
18. $\frac{1}{3}\log a - 6\log b + 2$

For problems 19 & 20 use the change of base formula and a calculator to find the value of each of the following.

19. $\log_{12} 35$
20. $\log_{\frac{2}{3}} 53$

For problems 21 - 23 sketch each of the given functions.

21. $g(x) = -\ln(x)$
22. $g(x) = \ln(x+5)$
23. $g(x) = \ln(x) - 4$

6.3 Solving Exponential Equations

Solve each of the following equations.

1. $6^{2x} = 6^{1-3x}$
2. $5^{1-x} = 25$
3. $8^{x^2} = 8^{3x+10}$
4. $7^{4-x} = 7^{4x}$
5. $2^{3x} = 10$
6. $7^{1-x} = 4^{3x+1}$
7. $9 = 10^{4+6x}$
8. $\mathbf{e}^{7+2x} - 3 = 0$
9. $\mathbf{e}^{4-7x} + 11 = 20$

6.4 Solving Logarithm Equations

Solve each of the following equations.

1. $\log_4\left(x^2 - 2x\right) = \log_4\left(5x - 12\right)$
2. $\log\left(6x\right) - \log\left(4 - x\right) = \log\left(3\right)$
3. $\ln\left(x\right) + \ln\left(x + 3\right) = \ln\left(20 - 5x\right)$
4. $\log_3\left(25 - x^2\right) = 2$
5. $\log_2\left(x + 1\right) - \log_2\left(2 - x\right) = 3$
6. $\log_4\left(-x\right) + \log_4\left(6 - x\right) = 2$
7. $\log\left(x\right) = 2 - \log\left(x - 21\right)$
8. $\ln\left(x - 1\right) = 1 + \ln\left(3x + 2\right)$
9. $2\log\left(x\right) - \log\left(7x - 1\right) = 0$

6.5 Applications

10. We have $10,000 to invest for 44 months. How much money will we have if we put the money into an account that has an annual interest rate of 5.5% and interest is compounded

    (a) quarterly **(b)** monthly **(c)** continuously

11. We are starting with $5000 and we're going to put it into an account that earns an annual interest rate of 12%. How long should we leave the money in the account in order to double our money if interest is compounded

    (a) quarterly **(b)** monthly **(c)** continuously

12. A population of bacteria initially has 250 present and in 5 days there will be 1600 bacteria present.

    (a) Determine the exponential growth equation for this population.

    (b) How long will it take for the population to grow from its initial population of 250 to a population of 2000?

13. We initially have 100 grams of a radioactive element and in 1250 years there will be 80 grams left.

    (a) Determine the exponential decay equation for this element.

    (b) How long will it take for half of the element to decay?

    (c) How long will it take until there is only 1 gram of the element left?